International Journal of Scientific & Engineering Research Volume 10, Issue 1, January-2019
ISSN 2229-5518

Identify the captivate And Anticaptivate Energy of Linear Polarization Of Electric Field

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January 3, 2019

Abstract

I study the captivate and anticaptivate energy of linear polarization of electric field. Derive plane wave of electric field from maxwell equations in free space and employing the approximation of linear variation method, I calculate the captivate and anticaptivate energy of linear polarization of electric field.The result indicate that, the captivate energy $E = \frac{\hbar^2\kappa^2}{6m}$ and anticaptivate energy is zero.

Key word: Electric Field;Linear olarization; Variation method;Captivate; Anti-captivate Energy

1 INTRODUCTION

Polarization is a fundamental property of light and a very important concept of physical optics. Not all sources of light are polarized; for instance, light from an ordinary light bulb is not polarized. In addition to unpolarized light, there is partially polarized light and totally polarized light. Light from a rainbow, reflected sunlight, and coherent laser light are examples of polarized light. There are three different types of polarization states: linear, circular and elliptical. Each of these commonly encountered states is characterized by a differing motion of the electric field vector with respect to the direction of propagation of the light wave. It is useful to be able to differentiate between the different types of polarization. Some common devices for measuring polarization are linear polarizers and retarders. Polaroid sunglasses are examples of po larizers. They block certain radiations such as glare from reflected sunlight. Polarizers are useful in obtaining and analyzing linear polarization. Retarders (wave plates) can alter the type of polar ization and/or rotate its direction. They are used in controlling and analyzing polarization states.

linear polarization is defined as polarization of an electromagnetic wave in which the electric vector at a fixed point in space remains pointing in a fixed direction, although varying in magnitude. There are two forms of linear polarization: vertical, where the electric field is perpendicular to the Earth's surface, and horizontal, where the electric field is parallel to the Earth's surface.The special case linearly polarized occur when phase difference between E_x and E_y is$\delta = n\pi$,n=0,1,2.... The phase relationship between these two components can

International Journal of Scientific & Engineering Research Volume 10, Issue 1, January-2019
ISSN 2229-5518

explain the different states of polarization.If the phase relationship is random, light is not polarized. If the phase relationship is random, but more of one component is present, the light is partially polarized. If the phase relationship is constant, the light is completely polarized.

2 Plane wave of electric field

Plane wave of electric field is the solution of the wave equation which derive from maxwell equations in free space(Faraday law and Amper's law).

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{1}$$

$$\nabla \times H = \frac{\partial D}{\partial t} \tag{2}$$

Then, multiply by curl

$$\nabla \times \nabla \times E = -\frac{\partial(\nabla \times B)}{\partial t} \tag{3}$$

Employing identity operator

$$\nabla \times \nabla \times E = \nabla E - \nabla^2 E \tag{4}$$

$$\nabla \times \nabla \times = -\nabla^2 E \tag{5}$$

$$\nabla^2 E = \frac{\partial}{\partial t}(\nabla \times B) \tag{6}$$

$$\nabla^2 E = \mu_0 \frac{\partial}{\partial t}(\frac{\partial D}{\partial t} \tag{7}$$

$$\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \tag{8}$$

The solution of wave equation is plane wave.so we can solve wave equation by applying the separation method,

$$E(r,t) = E(r)E(t) \tag{9}$$

$$E(t)\frac{\partial^2}{\partial r^2}E(r) = E(r)\frac{1}{c^2}\frac{\partial^2 E(t)}{\partial t^2} \tag{10}$$

multiply by $\frac{1}{E(r)E(t}$

$$\frac{1}{E(r)}\frac{\partial^2}{\partial r^2}E(r) = \frac{1}{E(t)}\frac{1}{c^2}\frac{\partial^2 E(t)}{\partial t^2} \tag{11}$$

$$\frac{1}{E(r)}\frac{\partial^2}{\partial r^2}E(r) = -\kappa^2 = \frac{1}{E(t)}\frac{1}{c^2}\frac{\partial^2 E(t)}{\partial t^2} \tag{12}$$

From this equation we have

$$\frac{1}{E(r)}\frac{\partial^2}{\partial r^2}E(r) = -\kappa^2 \tag{13}$$

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$$\frac{1}{E(t)}\frac{1}{c^2}\frac{\partial^2 E(t)}{\partial t^2} = -\kappa^2 \tag{14}$$

$$\frac{\partial^2}{\partial r^2}E(r) + \kappa^2 E(r) = 0 \tag{15}$$

$$\frac{\partial^2 E(t)}{\partial t^2} + \kappa^2 c^2 E(t) = 0 \tag{16}$$

This two equations are homogenous second ODE. let

$$E(r) = e^{\lambda r} \tag{17}$$

$$\ddot{E(r)} = \lambda^2 e^{\lambda r} \tag{18}$$

$$\lambda = \pm i\kappa \tag{19}$$

$$E(r) = Ae^{i\kappa r} + Be^{-i\kappa r} \tag{20}$$

similarly

$$E(t) = Ce^{i\omega t} + De^{-i\omega t} \tag{21}$$

$$E(r,t) = (Ae^{i\kappa r} + Be^{-i\kappa r})(Ce^{i\omega t} + De^{-i\omega t}) \tag{22}$$

$$E(r,t) = Ace^{i(\kappa r+\omega t)} + ADe^{i(\kappa r-\omega t)} + BCe^{-i(\kappa r-\omega t)} + BDe^{i(\kappa r+\omega t)} \tag{23}$$

Ignoring term 1 and 4,since not the wave function of particles.

$$E(r,t) = ADe^{i(\kappa r-\omega t)} + BCe^{-i(\kappa r-\omega t)} \tag{24}$$

using the Euler's rule

$$E(r,t) = ADcos(\kappa r-\omega t)+iADsin(\kappa r-\omega t)+BCcos(\kappa r-\omega t)-iBCsin(\kappa r-\omega t) \tag{25}$$

$$E(r,t) = (AD+BC)cos(\kappa r-\omega t) + i(AD-BC)sin(\kappa r-\omega t) \tag{26}$$

At AD=BC

$$E(x,y,t) = E_0cos(\kappa z-\omega t)i + E_0cos(\kappa z-\omega t)j \tag{27}$$

It is plane wave of electric field.

3 Linear variation method

The variation method allows us to obtain an approximation to the ground state energy of the system without solving the Schrödinger equation. The principle of the variation is say that the expectation value energy will be greater than the true value of energy (E_0)with the trial function(ψ).

$$\psi = \alpha_1\phi_1 + \alpha_2\phi_2....\alpha_n\phi_n \tag{28}$$

where ϕ_1 and ϕ_2 are basis function.
α_1 and α_2 mixing coefficient.

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The expectation value energy

$$\bar{E} = \frac{\int \psi H \psi d\tau}{\int \psi \psi d\tau}$$
$$\bar{E} = \frac{\int (\alpha_1\phi_1 + \alpha_2\phi_2) H (\alpha_1\phi_1 + \alpha_2\phi_2) d\tau}{\int (\alpha_1\phi_1 + \alpha_2\phi_2)(\alpha_1\phi_1 + \alpha_2\phi_2) d\tau}$$
$$\bar{E} = \frac{\alpha_1^2 \int \phi_1 H \phi_1 d\tau + \alpha_2^2 \int \phi_2 H \phi_2 d\tau + 2\alpha_1\alpha_2 \int \psi_1 H \psi_2 d\tau}{\alpha_1^2 \int \phi_1^2 d\tau + \alpha_2^2 \int \phi_2^2 d\tau + 2\alpha_1\alpha_2 \int \psi_1 \psi_2}$$
$$\bar{E} = \frac{\alpha_1^2 H_{11} + \alpha_2^2 H_{22} + 2\alpha_1\alpha_2 H_{12}}{\alpha_1^2 K_{11} + \alpha_2^2 K_{22} + 2\alpha_1\alpha_2 K_{12}}$$

Where $H_{11} = \int \phi_1 H \phi_1 d\tau$,$H_{22} = \int \phi_2 H \phi_2 d\tau$, $H_{12} = \int \phi_1 H \phi_2 d\tau = \int \phi_2 H \phi_1 d\tau$
$K_{11} = \int \phi_1 \phi_1 d\tau$, $K_{22} = \int \phi_2 \phi_2 d\tau$,$K_{12} = \int \phi_1 \phi_2 d\tau = \int \phi_2 \phi_1 d\tau$
For minimization of expectation value energy

$$\frac{\partial}{\partial \alpha_1}\bar{E} = \frac{\partial}{\partial \alpha_2}\bar{E} = 0 \tag{29}$$

$$\frac{\partial}{\partial \alpha_1}\bar{E} = \frac{\partial}{\partial \alpha_1}\left(\frac{\alpha_1^2 H_{11} + \alpha_2^2 H_{22} + 2\alpha_1\alpha_2 H_{12}}{\alpha_1^2 K_{11} + \alpha_2^2 K_{22} + 2\alpha_1\alpha_2 K_{12}}\right) \tag{30}$$

$$\alpha_1 H_{11} + \alpha_2 H_{12} - \bar{E}(\alpha_1 K_{11} + \alpha_2 K_{12}) \tag{31}$$

$$\alpha_1(H_{11} - \bar{E}K_{11}) + \alpha_2(H_{12} - \bar{E}K_{12}) \tag{32}$$

Similarly

$$\frac{\partial}{\partial \alpha_1}\bar{E} = \frac{\partial}{\partial \alpha_2}\left(\frac{\alpha_1^2 H_{11} + \alpha_2^2 H_{22} + 2\alpha_1\alpha_2 H_{12}}{\alpha_1^2 K_{11} + \alpha_2^2 K_{22} + 2\alpha_1\alpha_2 K_{12}}\right) \tag{33}$$

$$\alpha_2 H_{22} + \alpha_2 H_{12} - \bar{E}(\alpha_2 K_{22} + \alpha_2 K_{12}) \tag{34}$$

$$\alpha_1(H_{12} - \bar{E}K_{12}) + \alpha_2(H_{22} - \bar{E}K_{22}) \tag{35}$$

In matrix form

$$\begin{pmatrix} H_{11} - \bar{E}K_{11} & H_{12} - \bar{E}K_{12} \\ H_{12} - \bar{E}K_{12} & H_{22} - \bar{E}K_{22} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = 0 \tag{36}$$

$$\begin{pmatrix} H_{11} - \bar{E} & H_{12} - \bar{E}K_{12} \\ H_{12} - \bar{E}K_{12} & H_{22} - \bar{E} \end{pmatrix} = 0 \tag{37}$$

This matrix(eq 37) is secular equation
where
$K_{11} = K_{22}$=1

$$\begin{pmatrix} \frac{\hbar^2\kappa^2}{4m} - \bar{E} & \frac{\hbar^2\kappa^2}{4m} - \frac{\bar{E}}{2} \\ \frac{\hbar^2\kappa^2}{4m} - \frac{\bar{E}}{2} & \frac{\hbar^2\kappa^2}{4m} - \bar{E} \end{pmatrix} = 0 \tag{38}$$

The determination of this matrix is

$$\frac{3}{4}E^2 - \frac{\hbar^2\kappa^2}{4m}E = 0 \tag{39}$$

$$E_+ = \frac{\hbar^2\kappa^2}{6m} \tag{40}$$

This is captivate energy of electric field

$$E_- = 0 \tag{41}$$

This is anticaptivate energy of electric field

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4 conclusion

Linear polarization is polarization of an electromagnetic wave in which the electric vector at a fixed point in space remains pointing in a fixed direction. We calculate the wave equation and plane wave of the electric field from the maxwell equations in free space. Employing the linear variation method solved the captivate and anticaptivate energy of linear polarization of electric field. The result indicate that, the captivate energy $E = 2\pi^2 \frac{\hbar^2}{3m\lambda^2}$ and anticaptivate energy is zero. The captivate energy is inversily proportional to the square of the wavelength $E \sim \frac{1}{\lambda^2}$. When the particles oscillate together, they have only captivate(attractive) energy.But, When the particles in molecule, they may have both captivate(attractive) and anticaptivate energy.

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